LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

THIRD SEMESTER – NOVEMBER 2009

ST 3812 / 3809 - STOCHASTIC PROCESSES

Date & Time: 05/11/2009 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

PART-A

Answer all the questions: 10x2=20

1) Define a process with stationary and independent increments.

2) Define a Markov chain. Let X_1, X_2 ……be a sequence of independent random variables. Does the process $\{X_n\}$ form a Markov chain?

3) Suppose that customers arrive at a bank according to Poisson process with 3 per minute. What is the probability of getting 4 customers in 2 minutes?

4) What is the distribution of time between 2 successive arrivals in a Poisson process with parameters λ ? What is the distribution of the waiting time for k arrivals?

5) Define a renewal process.

6) Define a Super Martingale $\{X_n\}$ with respect to $\{Y_n\}$.

7) Obtain $E[X_1 + X_2 + \dots + X_N]$ when N is a random variable independent of X_i.

8) State Renewal theorem.

9) Explain linear growth with immigration.

10) Show that communication of states satisfies transitive relation.

PART-B

Answer any 5 questions: 5x8=40

11) Define the transition probability matrix and show that the transition probability matrix with the initial distribution completely determines the chain.

12) Let $S_n = X_1 + X_2 + \dots + X_n$ where X_1, X_2 ……are independent random variables. Let $S_0=0$. $Pr\{X_i=k\}=a_k$, $a_k \geq 0$ $\sum a_k=1$. Show that $\{S_n\}$ forms a Markov chain. Obtain the transition probability matrix.

13) Let the life time density of a renewal process be $f(x) = x\, e^{-X}$, $x>0$. Obtain M (t).

14) Obtain the expression for $P_n(t)$ in Yule process when X(0)=1.

15) Define i) recurrent state

ii) period of a state

Show that recurrence is a class property.

16) Obtain the Kolmogorov-backward differential equations for a birth and death process.

17) Let $P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$, $0 < a, b < 1$

Prove $P^n = \frac{1}{(a+b)}\{A + (1-a-b)^n B\}$,

where $A = \begin{bmatrix} b & a \\ b & a \end{bmatrix}$ $B = \begin{bmatrix} a & -a \\ -b & b \end{bmatrix}$

18) Let $\{X(t)\}$ be a Poisson process with parameter λ. Suppose each arrival is registered with probability p independent of other arrivals. Let $\{Y(t)\}$ be the process of registered arrivals .Show that Y(t) is also Poisson.

PART-C

Answer any 2 questions 2x20=40

19) a) Show that in a positive recurrent apeoridic class
$\lim_{n\to\infty} P_{jj}^{n} = л_j = \sum л_i P_{ij}$, $\sum л_i = 1$ and the л' s are uniquely determined.

b) Given the following one – step tpm, obtain the stationary distribution for the Markov chain after verifying the conditions required for a unique solution.

$$P= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

(10+10)

20) a) State the postulates of a Poisson process.

b) Derive an expression for $P_n(t)$ in a Poisson process.

c) Let $X_1(t)$ and $X_2(t)$ be two independent Poisson processes with parameters λ_1 and λ_2 respectively. Obtain the pdf of $X_1(t)$ given $X_1(t) + X_2(t) = n$

(5+10+5)

21) a) Obtain the expression for $E[S_{N(t)+1}]$ in a renewal process.

b) Find $P[N(t) \geq k]$ in a renewal process having lifetime density
$f(x) = \rho e^{-\rho(x-\delta)}$ for $x>\delta$

c) A continuous time Markov chain has two states labeled 0 and 1.The waiting time in state 0is exponential with parameter λ . The waiting timein state 1is exponential with parameter μ .Obtain the expression for $P_{00}(t)$.

(7+7+6)

22) a) Obtain the mean and the variance of branching process.

b) Show that for a branching process $X_n = m^{-n} Y_n$ is a martingale.

c) Let the distribution of number of offsprings be geometric with
$P_k = b(1-b)^k$, k=0, 1, 2,…., 0<b<1
Obtain the probability of extinction. (10+5+5)

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